PERGAMON

Solid State Communications 113 (2000) 257–261

solid state communications

www.elsevier.com/locate/ssc

The classical two-dimensional Ising model in the static fluctuation approximation

R.R. Nigmatullin[a,b], A.A. Khamzin[a], H.B. Ghassib[b,*]

[a]Theoretical Physics Department, Kazan State University, Kazan, 420008, Tatarstan, Russia
[b]Department of Physics, University of Jordan, Amman, Jordan

Received 7 October 1999; accepted 13 October 1999 by H. Eschrig

Abstract

The classical two-dimensional Ising model is studied in the framework of the so-called static fluctuation approximation. This approximation hinges on the replacement of the *square* of the local-field *operator* with its mean value. The physical implication is that the true quantum-mechanical spectrum of this operator is replaced with a distribution; the moments of this distribution as well as the expectation value of the operator are then calculated *self-consistently*. With this *single controllable* approximation, the full thermodynamics and the pair correlation function of the classical two-dimensional Ising model are determined for the regular infinite lattice. The possible application to ultrathin ferroelectric films is discussed.

Keywords: A. Ferroelectrics; A. Thin films; D. Heat capacity; D. Order–disorder effects; D. Phase transitions

1. Introduction

Recently [1] it has been suggested that the critical behavior of ultrathin Langmuir–Blodgett-deposited polymer (FLP) films could be explained, at least in principle, by invoking the *classical* two-dimensional Ising model. However, previous attempts in this framework [2–4] have invariably been plagued by one or more of the following drawbacks: (1) experimental boundary conditions have not been incorporated into the formalism; (2) no correlation functions have been calculated and (3) the thermodynamic picture involved (including magnetization and specific heat) has been incomplete.

Motivated, in part, by the desire to overcome these drawbacks, the present work puts forward a new approach, the so-called *static fluctuation approximation* (SFA) [5,6], that could ultimately evolve into a full-fledged theory for explaining the phenomena in question. This approach is based on a *single controllable* approximation—namely, the replacement of the square of the local-field operator by its mean value. The underlying physical picture is that the true quantum-mechanical spectrum of this operator is replaced with a distribution; the moments of this distribution as well as the expectation value of the operator are then calculated *self-consistently*. It turns out that this sole approximation is adequate for a *complete description* of the system with different boundary conditions.

That this is the case is shown in Section 2. Our results are presented and discussed in Section 3. Concluding remarks follow in Section 4.

2. Formalism

The Ising model is described by the Hamiltonian

$$H = -h\sum_f S_f^z - \tfrac{1}{2}\sum_{ff'} U_{ff'} S_f^z S_{f'}^z \equiv -\sum_f \sigma_f S_f^z. \qquad (1)$$

Here f is a node of a two-dimensional lattice; $U_{ff'}$ is the interaction between particles located at the nodes f and f'; S_f^z is the z-component of a classical spin-vector $\mathbf{S}_f$ (dipole) of unit length; h is the value of the applied external field;

$$\sigma_f \equiv h + \sum_{f'} U_{ff'} S_{f'}^z \qquad (2)$$

* Corresponding author. Fax: +962-6-5355511/5355522.
E-mail address: ghassib@rss.gov.jo (H.B. Ghassib)

0038-1098/00/$ - see front matter
PII: S0038-1098(99)00476-7